FREEZING OF THE THAWED ZONE AROUND A WELL IN FROZEN SOILS TAKING INTO ACCOUNT THE PRESSURE-DEPENDENCE OF THE TEMPERATURE OF FREEZING

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UDC 536.416

The effects of taking into account the drop in the freezing point of water in the calculation of the pressures of retrograde freezing, arising during idle time and temporary shut-down of wells in frozen soils, are analyzed.

One of the most dangerous complications in the operation of wells in frozen soils is the collapse of the casing columns caused by the pressures generated by retrograde freezing. They appear as a result of the increase, accompanying freezing, in the volume of the water-bearing mass forming during drilling or operation around the well [1]. The water-bearing mass can be the soil which melts during operation of the well or the filtrate of flushing liquid in the caverns forming at the time the well is drilled. The freezing of such a water-bearing mass can increase the pressure in the medium surrounding the well up to magnitudes at which the rheological properties of the soil and the pressure dependence of the phase-transition temperature become significant [2]. An increase in the pressure decreases the freezing point of water, which slows down the process of retrograde freezing. Relaxation of the pressure lowers the pressure level of retrograde freezing and thereby raises the freezing point according to the phase diagram for water, which accelerates the freezing process. This close interrelationship of the rheological properties of the soil and heat transfer in the process of retrograde freezing is analyzed in an axisymmetrical formulation of the problem, which we proposed in [2].

We shall study the basic equations which determine the temperature and pressure fields in the process of freezing of the thawed region forming around a well after the well is shut down. We shall separate three zones, differing by their thermophysical and physico-mechanical parameters. The first zone $R_0 < \bar{r} < \bar{s}$, corresponds to the thawed phase of the water-bearing mass. The second, $\bar{s} < \bar{r} < s_t$, corresponds to the mass which freezes during the process of retrograde freezing. The third zone, $\bar{r} > \bar{s}_t$, consists of the frozen soil which has not been subjected to thawing. We assume that the materials in all three zones are incompressible, the physical parameters of the second and third zone are equal, and the pressure distribution in the first zone is hydrostatic. These assumptions simplify the solution and given an upper limit for the pressures.

We shall write the equation of heat conduction for the thawed and frozen zones in the following form:

thawed zone

$$\frac{\partial\theta_1}{\partial t} = \varkappa_1\left(\frac{1}{r}\,\frac{\partial\theta_1}{\partial r} + \frac{\partial^2\theta_1}{\partial r^2}\right), \tag{1}$$

$$1 \leqslant r \leqslant s(t), \quad t_{\text{t}} \leqslant t < t_f,$$

$$\theta_1|_{r=s} = \theta_{\text{ph}}(t), \tag{2}$$

$$\left.\frac{\partial\theta_1}{\partial r}\right|_{r=1} = 0, \tag{3}$$

$$\theta_1|_{t=t_{\text{t}}} = \theta_{\text{t}}(r); \tag{4}$$

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 1, pp. 122-129, January, 1985. Original article submitted August 15, 1983.

0022-0841/85/4801-0101$09.50

frozen zone

$$\frac{\partial\theta_2}{\partial t}=\frac{1}{r}\frac{\partial\theta_2}{\partial r}+\frac{\partial^2\theta_2}{\partial r^2},\quad s(t)<r<\infty, \tag{5}$$

$$\theta_2|_{r=s}=\theta_{ph}\ \text{for}\ t_t\leqslant t\leqslant t_f, \tag{6'}$$

$$\left.\frac{\partial\theta_2}{\partial r}\right|_{r=1}=0\quad\text{for}\quad t\geqslant t_f, \tag{6''}$$

$$\theta_2|_{t=t_t}=\theta_t(r). \tag{7}$$

In view of the fact that the thermophysical properties of the second and third zones are assumed to be equal, a single problem of heat conduction with an index 2 is written for them.

The following conditions are satisfied on the phase-transition boundary r = s:

$$-\lambda_1\frac{\partial\theta_1}{\partial r}+\lambda_2\frac{\partial\theta_2}{\partial r}=\frac{ds}{dt}, \tag{8}$$

$$s|_{t=t_t}=s_t \tag{9}$$

Using Maxwell's model for the frozen state of the medium in the second and third zones, we obtain in accordance with [2] an expression for the pressure on the phase-transition boundary:

$$p=\alpha_v\left\{\left(1-\frac{s^2}{s_t^2}\right)(G_3-G_2)-\int\limits_{t_t}^{t}\left(1-\frac{s^2(\nu)}{s_t^2}\right)\left[\frac{G_3}{\tau_{r3}}\exp\left(\frac{\nu-t}{\tau_{r3}}\right)-\frac{G_2}{\tau_{r2}}\exp\left(\frac{\nu-t}{\tau_{r2}}\right)\right]d\nu\right\}. \tag{10}$$

We write the relationship between the pressure and the temperature of the phase transition in accordance with [3] in the form

$$p=p_a+A_1|\theta_{ph}T_0|-A_2|\theta_{ph}T_0|^2. \tag{11}$$

In the relations presented above and below we used the following dimensionless variables and parameters [2]:

$$\theta_i=T_i/T_0,\quad t=\tau a_2/R_0^2,\quad r=\bar{r}/R_0,\quad s=\bar{s}/R_0,$$

$$t_t=\tau_t a_2/R_0^2,\quad R=\bar{R}/R_0,\quad \theta_F=T_F/T_0,\quad \varkappa_1=a_1/a_2,$$

$$\tau_{ri}=\bar{\tau}_{ri}a_2/R_0^2,\quad \lambda_i=\bar{\lambda}_iT_0/(a_2\rho wl),\quad s_t=\bar{s}_t/R_0,$$

$$\theta_{ph}=T_{ph}/T_0,\quad G_i=\bar{G}_i/p_0,\quad p=\bar{p}/p_0,$$

$$p_a=\bar{p}_a/p_0,\quad W=\bar{W}/T_0,\quad t_f=\tau_f a_2/R_0^2,$$

where the values of the index i = 1, 2, and 3 correspond to the enumeration of the zones introduced above.

We solve the problem (1)-(11) by the integral method [4], used in [2] for the case of retrograde freezing without the pressure-dependence of the phase-transition temperature. We shall first examine the first stage of the process, $t_t \leqslant t \leqslant t_f$, when freezing is observed and both the thawed and frozen phases of the water-bearing mass are present. Integrating Eq. (1) with respect to the spatial coordinates, we find

$$\frac{d\Phi_1}{dt}-s\theta_{ph}\frac{ds}{dt}=\varkappa_1 s\left.\frac{\partial\theta_1}{\partial r}\right|_{r=s}, \tag{12}$$

where

$$\Phi_1=\int\limits_1^s r\theta_1 dr.$$

We seek the temperature profile in the thawed zone in the form

$$\theta_1=\frac{W\left(\ln\frac{r}{s}-r+s\right)+\theta_{ph}(r-\ln r-1)}{s-1-\ln s}. \tag{13}$$

This profile satisfies the boundary conditions (2) and (3), and the unknown function W(t) — the temperature of the well wall — entering into (13) is determined from the solution of the problem. Substituting (13) into (12), we arrive at the following ordinary differential equation:

$$\frac{\dot{s}R_s}{(s-1-\ln s)^2}+\frac{\dot{W}R_w+\dot{\theta}_{ph}R_\theta}{s-1-\ln s}-s\dot{s}\theta_{ph}=\frac{\varkappa_1(s-1)(\theta_{ph}-W)}{s-1-\ln s}, \tag{14}$$

where

$$R_s=\left(\frac{1}{s}-1\right)\left[\frac{s^3}{6}(2\theta_{ph}+W)-\frac{s^2}{2}\theta_{ph}\ln s-\frac{s^2}{4}(\theta_{ph}+W)+\right.$$

$$\left.+\frac{W}{2}(\ln s-s)+\frac{7W-\theta_{ph}}{12}\right]+(s-1-\ln s)\left[\frac{s^2}{2}(2\theta_{ph}+W)-s\ln s\theta_{ph}-\frac{s}{2}(2\theta_{ph}+W)+\frac{W}{2}\left(\frac{1}{s}-2\right)\right];$$

$$R_w=\frac{1}{12}(2s^3-3s^2+6\ln s-6s+7);$$

$$R_\theta=\frac{1}{12}(4s^3-6s^2\ln s-3s^2-1).$$

We seek the temperature profile in the frozen zone in the form

$$\theta_2=B_1\ln r+B_2;\quad s\leqslant r\leqslant R(t). \tag{15}$$

Here R(t) is the moving boundary of the thermal disturbance in the soil, which satisfies the condition

$$\theta_2|_{r=R}=\theta_F.$$

Substituting the profile (15) into the relation expressing the thermal balance in the frozen zone, we obtain

$$\frac{\theta_F}{4\ln^2\frac{R}{s}}\left[2(s\dot{s}-R\dot{R}\ln\frac{R}{s}-(s^2-R^2)\left(\frac{\dot{R}}{R}-\frac{\dot{s}}{s}\right)\right]+$$

$$+\dot{\theta}_{ph}\left(\frac{R^2-s^2}{4\ln\frac{R}{s}}-\frac{s^2}{2}\right)+\frac{\theta_{ph}}{4\ln^2\frac{R}{s}}\left[2(R\dot{R}-s\dot{s})\ln\frac{R}{s}-(R^2-s^2)\left(\frac{\dot{R}}{R}-\frac{\dot{s}}{s}\right)\right]=\frac{\theta_{ph}-\theta_F}{\ln\frac{R}{s}}. \tag{16}$$

Taking into account the form of the temperature profiles adopted in the thawed and frozen regions, we write Stefan's condition (8) as follows:

$$-\frac{\lambda_1\left(\frac{1}{s}-1\right)(W-\theta_{ph})}{s-1-\ln s}+\frac{\lambda_2(\theta_F-\theta_{ph})}{s\ln\frac{R}{s}}=\dot{s}. \tag{17}$$

To simplify the numerical solution of the problem, we introduced the following auxiliary functions:

$$J_i=\int_{t_t}^{t}\left(1-\frac{s^2(\nu)}{s_t^2}\right)\exp\left(\frac{\nu-t}{\tau_{pi}}\right)d\nu;\quad i=2,\ 3, \tag{18}$$

which satisfy first-order differential equations of the form

$$\dot{J}_i=1-\frac{s^2(t)}{s_t^2}-\frac{J_i}{\tau_{ri}};\quad i=2,\ 3. \tag{19}$$

Differentiating relation (11) with respect to time, we obtain, taking into account (10) and (18), the equation

$$\dot{\theta}_{ph} = -\frac{\alpha_v}{T_0(A_1 + 2A_2T_0\theta_{ph})}\left[-\frac{2s\dot{s}}{s_t^2}(G_3 - G_2) - \left(1 - \frac{s^2}{s_t^2}\right)\left(\frac{G_3}{\tau_{r3}} - \frac{G_2}{\tau_{r2}}\right) + \frac{G_3J_3}{\tau_{r3}^2} - \frac{G_2J_2}{\tau_{r2}^2}\right]. \quad (20)$$

Thus, six ordinary first-order differential equations (14), (16), (17), (19), and (20) with the initial conditions

$$s|_{t=t_t} = s_t,\quad R|_{t=t_t} = R_t,\quad W|_{t=t_t} = W_t, \qquad (21)$$
$$\theta_{ph}|_{t=t_t} = 0,\quad J_2|_{t=t_t} = 0,\quad J_3|_{t=t_t} = 0.$$

have been obtained for the six unknown functions s, R, W, θ_{ph}, J_2, J_3. Here s_t, R_t, W_t are the values of the radius of the boundaries of thawing and of the thermal effect and temperature of the well wall at the time t_t at which heat stops flowing from the well into the surrounding medium. The quantities s_t, R_t, W_t are determined from the solution of the problem of heating of the frozen soil surrounding the working well [2].

Equations (14), (17), and (20) have a singularity at the point s = 1, i.e., at the time $t = t_f$ that the freezing front reaches the well wall. In the vicinity of the singular point

$$t^* \leqslant t \leqslant t_f,\quad 1 \leqslant s \leqslant 1 + \delta$$

the indicated equations can be represented in the following form

$$\dot{\theta}_{ph} + \frac{2}{3}\dot{W} = \frac{2\varkappa_1(\theta_{ph} - W)}{(s-1)^2} + \frac{2}{3}\dot{s}\frac{(\theta_{ph} - W)}{s-1}, \qquad (22)$$

$$\dot{s} = \frac{2\lambda_1(W - \theta_{ph})}{s-1} + \frac{\lambda_2(\theta_F - \theta_{ph})}{\ln R}, \qquad (23)$$

$$\dot{\theta}_{ph} = -\frac{\alpha_v}{T_0(A_1 + 2A_2T_0\theta_{ph})}\left[-\frac{2\dot{s}}{s_t^2}(G_3 - G_2) + \frac{J_3G_3}{\tau_{r3}^2} - \frac{J_2G_2}{\tau_{r2}^2}\right]. \qquad (24)$$

The system of Eqs. (22)-(24) admits a self-similar solution of the form

$$s = 1 + c_1(t - t_f), \qquad (25)$$
$$\theta_{ph} = \theta_f + c(t - t_f) + c_2(t - t_f)^2, \qquad (26)$$
$$W = \theta_f + c(t - t_f) + c_3(t - t_f)^2, \qquad (27)$$

where c, c_1, c_2, c_3 are coefficients to be determined; $\theta_f = \theta_{ph}(t_f) = W(t_f)$.

We introduce the following notation:

$$s^* = s(t^*),\quad \theta^*_{ph} = \theta_{ph}(t^*),\quad W^* = W(t^*);$$
$$R^* = R(t^*),\quad J_i^* = J_i(t^*),\quad I^* = \frac{J_3^*G_3}{\tau_{r3}^2} - \frac{J_2^*G_2}{\tau_{r2}^2}.$$

The values of these quantities can be obtained by numerically solving the equations introduced above in the time interval $[t_t, t^*]$. Substituting the functions (25)-(27) into the system (22)-(24), we can obtain a system of algebraic equations for the unknown coefficients:

$$c_1 = \lambda_2\frac{\theta_F - \theta_f}{\ln R^*}, \qquad (28)$$

$$\frac{2\varkappa_1 c_4}{c_1^2} = -\frac{5}{3}\,\frac{\alpha_v(Gc_1 + I^*)}{T_0(A_1 + 2A_2T_0\theta^*_{ph})}, \qquad (29)$$

$$c_4(t^* - t_f)^2 = \theta^*_{ph} - W^*, \qquad (30)$$

$$c_1 = (s^* - 1)/(t^* - t_f), \qquad (31)$$

where $c_4 = c_2 - c_3$; $G = -2(G_3 - G_2)/s_t^2$. Solving the system (28)-(31), we find the dimensionless time t_f at which the thawed zone disappears and the values of the phase-transition and wall temperatures at this time.

After the thawed zone has vanished ($t > t_f$), the pressure and temperature fields continued to evolve. The temperature field is described by Eq. (5) with the boundary condition (6") and the initial condition at $t = t_f$, following from the continuity of the temperature at this time. To obtain an approximate solution of the problem at the stage $t > t_f$, we assume that the temperature profile has the form

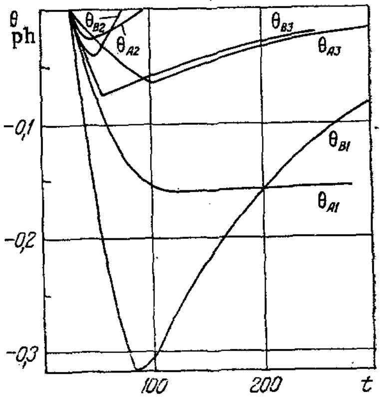
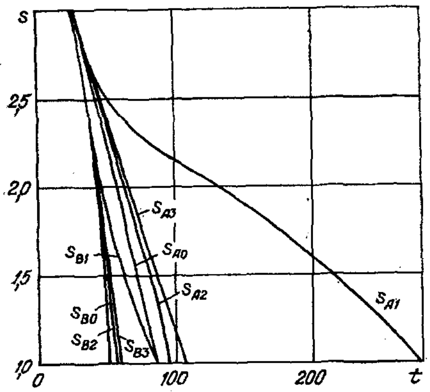


Fig. 1. Behavior of the phase-transition temperature θ_{ph} as a function of time (θ_{ph}, t are dimensionless).



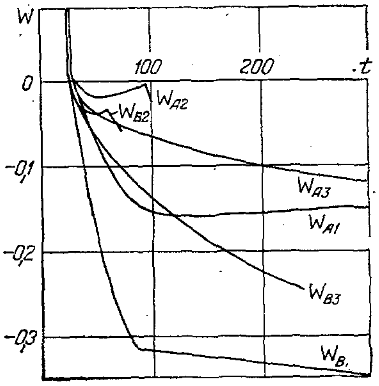


Fig. 2 Fig. 3

Fig. 2. Dynamics of the motion of the freezing front s(t) (s, t are dimensionless).

Fig. 3. Behavior of the temperature of the well wall W as a function of time (W, t are dimensionless).

$$\theta_2 = \theta_F + \frac{W - \theta_F}{2}\left[1 + \cos\left(\pi\,\frac{r-1}{R-1}\right)\right]. \tag{32}$$

This distribution satisfies the boundary condition (6") and the condition that the temperature gradient vanish at the radius of the thermal effect. From the condition that the heat content of the soil remain constant

$$\int_1^R r\theta_2 dr = K \text{ for } \quad t > t_f$$

we obtain a relation between the radius of the thermal effect and the temperature at the wall of the well:

$$\frac{W - \theta_F}{W_f - \theta_F} = \frac{(R_f^2 - 1)\pi^2 - 4(R_f - 1)^2}{(R^2 - 1)\pi^2 - 4(R - 1)^2}, \tag{33}$$

where $W_f = W(t_f)$, $R_f = R(t_f)$.

Relation (33) together with the law for R(t) [2]

$$t = \frac{1}{12}\left(R^2 + R - \frac{6R\ln R}{R-1} + 4\right) \tag{34}$$

describe the behavior of the temperature W(t) after the thawed zone has vanished. The pressure at the wall of the well for $t > t_f$ is determined by Eqs. (10) and (19), where it is necessary to set s(t) = 1.

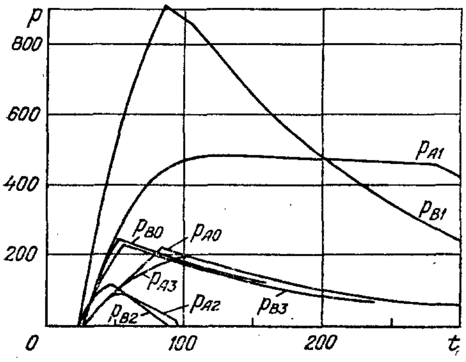


Fig. 4. Results of the calculation of the pressure p(t) (p, t are dimensionless).

The proposed method for calculating the temperature and pressure fields on the casing column was implemented on the Nord computer, for which a Fortran-V program was written. The following parameters were used in the calculations, in accordance with [2], for all variants: $\bar{\lambda}_1$ = 1.54 kcal/m·h·deg; $\bar{\lambda}_2$ = 1.8 kcal/m·h·deg; $\tilde{\lambda}_1$ = 0.06 kcal/m·h·deg; a_1 = $2.49 \cdot 10^{-3}$ m²/h; a_2 = $2.92 \cdot 10^{-3}$ m²/h; T_0 = 25°C; w = 0.159; α_y = 0.03; ρ = 1400 kg/m³; l = 80 kcal/kg; R_0 = 0.14 m; R_1 = 0.1304 m; $\bar{G}_2 = 4 \cdot 10^4$ kgf/cm²; $\bar{G}_3 = 5 \cdot 10^4$ kgf/cm²; $p_0 = \bar{p}_a$ = 1 kgf/cm².

The results of the calculations are presented in Figs. 1-4, where the curves corresponding to the variants have a double letter-number subscript. Calculations were performed for two series of variants, differing by the value of the temperature of the frozen soil T_F. For T_F = −5°C the curves in the figures are labelled by the index A and the series T_F = −10°C corresponds to the index B. The number indices on the curves 1-3 correspond to one of the following values of the relaxation time in order of enumeration: τ_{r3}/τ_{r2} = 1000/100; 500/1000; 1000/1000 (h). The index 0 corresponds to the solution ignoring the dependence $\theta_{ph}(p)$ for the case τ_{r3}/τ_{r2} = 1000/1000. An analysis of the computational results shows that taking into account the pressure dependence of the phase-transition temperatures decreases the rate of freezing and therefore also lowers the values of the pressures and temperatures of the phase transition. In connection with this, the acceleration of the freezing front accompanying the drop in T_F will be less distinct than when the dependence $\theta_{ph}(p)$ is ignored. The nature of the process is strongly affected by the physicomechanical parameters. This is manifested as an increase in the magnitudes of the pressure accompanying the increase in the viscosity $\mu_3 = G_2\tau_{r3}$ of the third zone compared to the corresponding value for the second zone. Depending on the ratio of the viscosities of these zones, the freezing can be accompanied by a continuous increase in pressure up to the moment of complete freezing or the pressures reach a maximum value in the time interval $[t_t, t_f]$. In the case $\mu_3 < \mu_2$ the pressures relax much more rapidly than for $\mu_3 > \mu_2$; in addition, the pressures increase when the ratio τ_{r3}/τ_{r2} increases.

We emphasize also the qualitative difference between the results of the calculation based on the viscoelastic model for the growth of the pressures studied here and the results obtained with the elastic and elastoplastic models [5]. This difference consists of the continuous relaxation of the pressures of retrograde freezing, manifested in the existence of their maxima at the end of the time interval, which for the elastic and elastoplastic models is reached over an infinitely long time.

NOTATION

τ, time; $\bar{r}$, radial coordinate; $\bar{s}$, radius of freezing; $\bar{s}_t$, initial radius of the thawed zone; τ_{ri}, relaxation times; T_i, T_0, temperature and its scale; R_0, radius of the well; T_{ph}, T_F, temperatures of the phase transition and of the initial frozen soil; a_1, a_2, coefficients of thermal diffusivity of the thawed and frozen zones; $\bar{\lambda}_1$, $\bar{\lambda}_2$, coefficients of thermal conductivity of the thawed and frozen zones; $\bar{W}$, temperature of the well wall; $\bar{G}_i$, elastic shear modulus; $\bar{p}$, p_0, pressure and its scale; $\bar{p}_a$, atmospheric pressure; and $\bar{R}$, radius of the thermal disturbance.

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A CLASS OF PLANE SELF-SIMILAR MOTIONS OF A NON-NEWTONIAN FLUID WITH NONLINEAR THERMOPHYSICAL PROPERTIES

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UDC 532.517:536.24

The plane self-similar solution of a set of complete equations for the dynamics of a nonlinearly viscous fluid and the energy equation is obtained analytically with the temperature dependence of the transfer coefficients taken into account.

1. INITIAL EQUATIONS AND NEW INDEPENDENT VARIABLES

We take the generalized Z. P. Shul'man model of a nonlinearly viscoplastic incompressible fluid as a basis and we write the equations of two-dimensional plane nonstationary motion, the continuity and heat balance equations [1]:

$$u_t + (p/\rho + u^2 - \tau_{11}/\rho)_x + (uv - \tau_{12}/\rho)_y = 0, \tag{1}$$

$$v_t + (uv - \tau_{12}/\rho)_x + (p/\rho + v^2 - \tau_{22}/\rho)_y = 0, \tag{2}$$

$$u_x + v_y = 0, \quad \rho \equiv \mathrm{const}, \tag{3}$$

$$\rho c_p (T_t + uT_x + vT_y) = (\lambda T_x)_x + (\lambda T_y)_y + A^2 B, \tag{4}$$

$$\tau_{11} = 2Bu_x, \quad \tau_{12} = \tau_{21} = B(u_y + v_x), \quad \tau_{22} = 2Bv_y, \tag{5}$$

$$A = [2u_x^2 + 2v_y^2 + (u_y + v_x)^2]^{\frac{1}{2}}, \quad B = [\tau_0^{\frac{1}{n}} A^{-\frac{1}{m}} + \mu^{\frac{1}{m}}]^n A^{\frac{n}{m}-1},$$

$$\mu = \mu(T), \quad \lambda = \lambda(T), \quad c_p = c_p(T).$$

We here assume p differentiable with respect to x, y, t and u, v, T twice differentiable with respect to x, y and once with respect to t. All these derivatives as well as the second mixed derivatives of the functions p, u, v, T in the arguments x, y, t are considered continuous in the space–time domain under consideration.

Equation (2) can be satisfied by taking

$$v = -\xi_y, \quad uv - \frac{\tau_{12}}{\rho} = \eta_y, \quad \frac{p - \tau_{22}}{\rho} + v^2 = \xi_t - \eta_x.$$

We substitute the expression p from the last formula into (1), regroup the terms therein by using the equality $\xi_{tx} = \xi_{xt}$ and satisfy the equation obtained as follows

Gomel' Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 1, pp. 129-136, January, 1985. Original article submitted September 26, 1983.

0022-0841/85/4801-0107$09.50